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COMPARATIVE ANALYSIS OF METHODS FOR ASSESSING RETENTION EXCEEDANCE PROBABILITY IN MOTOR CASCO INSURANCE: THE CASE OF BULGARIA

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The study explores some methods used to assess risk and determine optimal retention levels in motor casco insurance, specifically focusing on comparing the three statistical techniques: Chebyshev's Inequality, the Monte Carlo Simulation, and Normal Distribution. By utilizing historical claims data from the Bulgarian insurance market published by the Financial Supervision Commission, the study investigates the probability of the claim exceeding retention thresholds and compares the accuracy and precision of each method. While Chebyshev's inequality provides a conservative estimate, the Monte Carlo simulation offers a probabilistic approach that models various outcomes, whereas normal distribution assumes a symmetrical loss pattern. The research aims to identify which method offers the most reliable estimation for setting retention levels in motor casco insurance. By evaluating the accuracy of each technique against real claims data, the study aims to inform insurers about the approach which optimizes their risk management decisions best. The research shows that the Monte Carlo simulation offers the most accurate and reliable estimates for motor casco retention decisions due to its flexibility in modelling various loss scenarios.

Keywords: motor casco insurance, direct insurer's retention, Chebyshev's Inequality, Monte Carlo Simulation, Normal Distribution method

JEL Classification: G22

INTRODUCTION

Motor casco insurance is an essential component of the insurance sector, not only for the insurers but also for society at large. It covers vehicles against damage caused by accidents, theft, or natural disasters, providing a safety net for vehicle owners. For individual policyholders, having a motor casco policy means peace of mind, knowing that they are financially protected in the event of unexpected damages. This coverage is particularly valuable in a society with a growing number of vehicles, where accidents or loss can result in high financial costs.

The insurance policy considered is a voluntary, comprehensive vehicle insurance that covers damages

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to the policyholder's own vehicle, unlike third-party liability insurance, which only covers damages caused to others. It typically protects against risks such as accidental damage, theft, natural disasters (e.g. storms, floods), fire, and glass breakage. Insurers assess risk based on factors like the vehicle type and age, the driver's profile, and the location. These elements help determine premiums by estimating the likelihood and potential cost of claims. Claims frequency is generally moderate, with frequent minor claims (e.g. scratches) and less frequent but severe events (e.g. theft, total loss). Frequency is influenced by the driver's behavior and road conditions, while severity depends on the nature of the incident itself.

The importance of motor casco insurance extends beyond individual benefits. Broader social benefits include reducing the financial burden on the state and on other individuals in case of accidents or damages. It helps maintain the stability of the automotive sector and ensures that owners and operators can repair or replace vehicles quickly, thus contributing to the overall economic efficiency, which is especially important in a rapidly evolving economy such as Bulgarian, where a significant proportion of the population relies on personal vehicles for transportation.

Motor casco insurance plays a significant role on Bulgaria's non-life insurance market. As of 2023, motor casco represents 29% of the gross premium income, placing it in the second position after the obligatory motor third-party liability insurance, which accounts for 40% of the entire market. Simultaneously, the gross claims incurred related to motor casco insurance amounted to BGN 410,536,263, accounting for 26% of all non-life insurance claims. These figures demonstrate both the market share and the critical importance of motor casco insurance in the country's broader insurance landscape.

One of the key tasks for insurance companies, particularly in motor casco insurance, is determining how much risk they are willing to retain and how much should be transferred to a reinsurer. This decision is far from simple and requires a careful evaluation of various factors. For example, the specifics of motor casco insurance - which includes coverage for theft, damage from natural events, and accidents - present particular challenges. The frequency and severity of claims can vary greatly depending on the type of vehicle, geographic location, and other risk factors.

Insurance companies must also consider their risk appetite, which refers to the level of risk they are prepared to take on in pursuit of profit. This is influenced by the factors such as financial stability, historical loss experience, and claims management capabilities. Historical loss experience plays a crucial role in assessing potential future claims. If an insurer has faced frequent large claims in the past, they might opt for a lower retention threshold to limit their exposure. On the other hand, insurers with a track record of low loss frequency may feel comfortable retaining a higher share of risk.

One of the most effective ways for insurance companies to determine appropriate retention levels is by analyzing past claims data. By examining paid losses related to motor casco insurance, insurers can gain valuable insights into their historical loss experience, which may enable them to make informed decisions on future retention strategies. This data allows insurers to identify trends and patterns in claims frequency and severity, which can help predict the potential for future claims. Armed with this information, insurers can determine the level of risk they are comfortable retaining and make informed decisions on how much risk to transfer to reinsurers.

Given the complexities involved in determining the optimal retention level, insurers frequently rely on various mathematical and statistical methods to assess risk, manage exposure, and optimize retention strategies. These methods help insurers estimate the retention threshold exceedance probability and understand the likelihood of large claims that could impact their financial stability. Among the methods most commonly used for such assessments are Chebyshev's inequality, the Monte Carlo simulation, and normal distribution.

While distinct in their approaches, these methods all aim to help insurers quantify the risks they face and establish the retention levels that balance profitability with financial stability. Given the complexities involved in setting retention levels, insurers are continually refining these models and exploring new techniques to improve their risk management strategies.

The primary objective of this research is to compare the accuracy and precision of different methods for estimating the probability of a claim exceeding the retention threshold in motor casco. Specifically, Chebyshev's inequality, the Monte Carlo simulation, and normal distribution will be applied to a common dataset so as to assess how closely each method estimates the likelihood of a loss surpassing the insurer's retention level. By evaluating the performance of these methods, the research goal is to identify which approach provides the most reliable and precise estimation for setting appropriate retention levels in the context of motor casco risks. Valuating the "accuracy and precision" of each method will be done using the historical loss data related to motor casco insurance published by the Financial Supervision Commission (https://www.fsc.bg/en/) in Bulgaria.

Based on this, the research hypothesis here is that the Monte Carlo simulation provides a more accurate and precise estimate of the retention threshold exceedance probability in motor casco insurance compared to the traditional methods such as Chebyshev's inequality and normal distribution, which is due to the ability of the Monte Carlo simulation to model numerous possible outcomes based on historical loss data, offering a more detailed and flexible approach to risk assessment. The hypothesis will be tested by comparing the results obtained from each method and analyzing their alignment with the observed claims data, thereby assessing which method most closely predicts the actual risk of exceeding retention. With the intention to support the statement that the Monte Carlo simulation will provide a more accurate and precise estimate, sensitivity analysis will be performed in order to validate the robustness of the results and assess how changes in the key assumptions, such as the mean loss, impact the retention threshold exceedance probability. The theoretical and methodological instruments applied include probability theory, risk modelling, and statistical simulation, with a sensitivity analysis performed on the official motor casco claims data.

The paper is structured into a few sections. Following Section 1, Section 2 is a review of the relevant literature on risk assessment and retention modelling. Section 3 presents the data used in the analysis and outlines the methodological framework, focusing on Chebyshev's inequality, the Monte Carlo simulation, and normal distribution. Section 4 discusses the results obtained by applying the selected methods and interprets their implications. Finally, in Section 5, conclusions are given by summarizing the key findings and suggesting directions for future research.

LITERATURE REVIEW

Research on the optimal motor casco insurance retention, reinsurance, and related distributions is limited, and few studies address this complex problem directly.

An in-depth examination of self-retention in property insurance, including motor casco, has been presented in several studies. In V. K. Kaishev's study (2004), the reinsurance contract is modelled under an excess of loss (XL) treaty (Kaishev, 2004). The goal is to maximize the joint survival probability of both the cedent (direct insurer) and the reinsurer. V. K. Kaishev (2004) calculates survival probabilities for both parties and determines the optimal retention that minimizes the difference between their survival probabilities. The study compares this approach with quota share reinsurance, suggesting that a balance between retention and reinsurance is crucial for optimal outcomes.

F. Glineur and J. Walhin (2006) apply convex optimization techniques to confirm de Finetti's results on proportional reinsurance. They extend these findings to variable quota share and surplus reinsurance, challenging the optimality of these approaches and providing deeper insights into how retention affects reinsurance decisions. A. Tsanakas and P. Millossovich (2015) focus on the sensitivity and robustness of optimal retention calculations under various distributional assumptions, including normal and non-normal scenarios.

S. Li, Z. Zhu and J. Peng (2022) explore an optimal stop-loss reinsurance model in uncertain claim environments. The results of their study reveals how retention should be determined based on risk measures such as value-at-risk (VaR), which can guide insurers in volatile conditions. J. Cai and K. Tan (2007) develop two optimization criteria based on value-at-risk (VaR) and conditional tail expectation (CTE) so as to determine optimal retention levels in stop-loss reinsurance. They demonstrate that, when optimal solutions exist, both criteria yield the same retention value, although the CTE-based method is generally more applicable due to less restrictive conditions.

L. Noviyanti, A. Z. Soleh, A. Chadidjah and H. A. Rusyda (2018) explore different retention models in quota-share reinsurance and also examine possible retention optimization in Indonesia. Using bivariate lognormal and exponential distributions alongside risk measures like VaR, expected shortfall (ES), and the minimum variance (MV), the authors aim to minimize risks, while simultaneously ensuring financial stability and regulatory compliance. A. Z. Soleh, L. Noviyanti and I. Nurrahmawati (2015) discuss how stop-loss reinsurance can reduce risk exposure by optimizing retention in automobile insurance. Their study models risk using compound Poisson and lognormal distributions, showing that retention can be optimized so as to minimize VaR.

In scenarios with incomplete data, X. Hu, H. Yang and L. Zhang (2015) propose a distribution-free approach to optimize retention in stop-loss reinsurance contracts. This method helps insurers minimize risk, even with incomplete information.

The study by Y. Kroll and D. Nue (1991) introduces a portfolio management approach to retention. This framework analyzes alternative management goals and offers testable implications for setting optimal proportional reinsurance retention levels, helping insurers balance risk and profitability.

More recent studies have introduced advanced analytical tools and empirical investigations into the assessment of retention in motor casco insurance. L. Fu and H. Wang (2021) investigate attrition using survival analysis to distinguish mid-term cancellations from nonrenewal. By incorporating time-dependent macroeconomic variables into retention modelling, they introduce a more responsive approach for insurers to anticipate and manage risk exposure over time. The paper by R. Erusalimov and N. Iliev (2022) offers an empirical contribution by analyzing how the COVID-19 pandemic affected motor casco insurance in Bulgaria. Their study reveals how external shocks alter premium income, claims frequency, and indemnity payouts, thus influencing retention behavior and risk levels during crisis periods. Finally, M. Leiria, E. Rebelo and N. deMatos (2021) explore the role of intermediary loyalty programs and customer behavior in motor insurance cancellations. Their findings, based on logistic regression, show that intermediary involvement and payment methods significantly affect retention probability, adding a behavioral dimension to otherwise quantitatively modelled risk frameworks.

In this context, additional theoretical grounding is provided by research addressing capital allocation and collective risk modeling in non-life insurance. J. Kočović, M. Mitrašević, M. Kočović and M. Jovović (2011) examine the challenges of capital allocation in insurance companies, emphasizing the fact that the adequacy of allocation depends on acknowledging the inverse relationship between risk and capital, as well as the diversification effects that shape solvency and performance outcomes. Their findings highlight the necessity of selecting allocation and risk assessment techniques consistent with insurers' overarching objective of value maximization - the principle that also underpins the determination of optimal retention levels. Building on this theoretical perspective, Z. Djuric (2013) develops a collective risk model for non-life insurance, conceptualizing claim frequency and severity as stochastic processes. This framework provides a rigorous mathematical basis for simulating claim variability and estimating the probability of exceeding predefined retention thresholds, thereby complementing the methodological approach adopted in the present study.

Moreover, recent studies have expanded the application of the Monte Carlo simulation in the analysis of motor casco retention. L. Xiong and D. Hong (2020) demonstrate the fact that the Monte Carlo methods effectively predict solvency and model risk scenarios in captive insurance by simulating historical loss data, closely aligning with probabilistic retention evaluation. T. Heinrich, J. Sabuco and J. Farmer (2021) apply industry-wide simulation models addressing the risk correlation issues pertinent for estimating the probabilities of retention exceedance, thus contributing valuable perspectives on portfolio-level risk in motor casco insurance. B. Bulut Karageyik and Ş. Şahin (2017) propose the excess-of-loss criterion approach for the optimal retention calculation, emphasizing the varying premium and ruin considerations that offer a complementary empirical lens for motor casco retention frameworks. E. Roos, R. Brekelmans, W. van Eekelen, D. den Hertog and J. S. H. van Leeuwaarden (2022) advance theoretical underpinnings by developing tight tail probability bounds extending Chebyshev's Inequality, furnishing a more refined basis for conservative risk estimation in insurance retention modelling. M. Covrig and D. Badea (2017) employ generalized linear models for claim frequency analysis in motor insurance, thereby corroborating the use of normal distribution and other statistical techniques in retention risk assessment. B. Shahriar and S. M. M. Ahmadi (2008) introduce a novel methodology for determining optimal reinsurance retention by minimizing value-at-risk (VaR) via the Monte Carlo simulation, with a focus on capital adequacy and potential loss mitigation. Their empirical application to Mellat Insurance Company's portfolio demonstrates that the optimal retention levels are approximately 58%, BGN 35,845 million and BGN 18,937 million under quota share, excess of loss, and surplus reinsurance contracts, respectively. Using Nigerian motor insurance data, O. Chukwudum (2019) applies the Monte Carlo simulation combined with Extreme Value Theory to model large claims frequency and severity, enabling optimized reinsurance retention decisions using the improved estimation of capital requirements and excess-of-loss pricing.

Together, these recent contributions extend traditional retention theory into contemporary, data-driven practice. Simultaneously, they provide a deeper understanding of the multifactorial nature of retention probability assessment in motor casco insurance - spanning statistical modelling, empirical analysis, and regulatory framing.

DATA AND METHODOLOGY

Data

The analysis of a number of claims and insurance companies' paid losses on motor casco insurance in the Bulgarian insurance market covers the period from 2018 to 2022 and is carried out based on the official statistics published by the Financial Supervision Commission and available on its website (https://www.fsc.bg/en/). The reason for the five-year period chosen is a lack of the officially published market data for a longer period. The data are shown in Bulgaria's official currency - the Bulgarian Lev (BGN). Bulgaria has been on the currency board since 1997, and the exchange rate of the Bulgarian Lev (BGN) is fixed against the exchange rate of the Euro. BGN 1 is exchanged for EUR 0.511292.

The data used in this study are aggregated across the non-life insurance companies operating in Bulgaria that offer motor casco insurance. As of the end of the analyzed five-year period (2018-2022), 17 out of the 22 licensed non-life insurers in the country provided motor casco coverage, according to the official statistics published by the Financial Supervision Commission. The use of aggregated data ensures a comprehensive market-level view, which is appropriate for analyzing trends and assessing the retention threshold exceedance probability in a generalized context.

Methodology

This study employs three distinct methods to estimate the probability of exceeding a given retention threshold in the context of motor casco insurance: Chebyshev's inequality, the Monte Carlo simulation, and normal distribution. These methods were selected based on their compatibility with the available official data, which consist exclusively of the aggregated and anonymized historical loss records published by the Financial Supervision Commission. While modern actuarial and data-driven techniques such as Generalized Linear Models (GLMs), Usage-Based Insurance (UBI), Dynamic Pricing, Machine Learning, Bayesian Methods, and Catastrophe Modelling are increasingly used in the insurance industry for pricing and risk assessment, their direct applicability to estimating optimal retention levels, particularly in the context of this research, is limited. Most of these methods are primarily designed to assess individual policyholder risk and set premiums, requiring detailed, real-time, or proprietary internal data such as telematics, detailed claims histories, and dynamic behavioral metrics. For instance, GLMs and machine learning models are commonly employed to predict claims frequency and severity at the individual level, while UBI and dynamic pricing rely on the continuous monitoring of driving behavior, and catastrophe models necessitate high-resolution exposure data. In contrast, the current study is exclusively based on the officially published aggregate data, which constrains the use of such advanced methodologies.

While certain elements of these approaches may indirectly inform strategic decisions around reinsurance and retention (e.g. through portfolio-level risk modelling), their effective application demands the data inputs that are not publicly available. Consequently, the research employs alternative methods - Chebyshev's inequality, the Monte Carlo simulation, and the normal distribution approach - which are robust to data limitations and more appropriate for estimating the probability of retention exceedance using the available data.

Each method applied in the study provides a unique approach, based on different assumptions about the distribution of losses. Below is an explanation of how each method is applied in order to estimate the upper bound probability of retention exceedance.

Chebyshev's Inequality is a non-parametric method that provides an upper bound on the probability of an event occurring beyond a certain threshold. This method is particularly useful in situations where the distribution of losses is unknown or not well-behaved, i.e. it may be skewed or have heavy tails. Skewness refers to the asymmetry of a distribution, where data is not evenly spread around the mean; if a distribution has a longer tail on the right, it is positively skewed, and if it has a longer tail on the left, it is negatively skewed. Heavy tails describe a distribution where extreme values (outliers) occur more frequently than they would in a normal distribution, meaning the probability of large deviations from the mean is higher, leading to a slower decay in the tail of the distribution. The method guarantees an upper bound on the probability of exceeding the threshold, regardless of the distribution itself.

$$P(|X - \mu| \geq k\sigma) \leq \frac{1}{k^2} \quad (1)$$

where:

X is the individual loss,

μ is the mean loss,

σ is the standard deviation of the losses, and

k is the number of the standard deviations from the mean.

In this research, Chebyshev's Inequality will be used to calculate the upper bound probability that the individual loss X will exceed the chosen retention threshold T. The threshold T is defined as:

$$T = \mu + k\sigma \quad (2)$$

where k is the multiple of the standard deviation. To calculate the probability of exceeding the threshold using Chebyshev's inequality:

1. Determine the number of the standard deviations k: Calculate how far the retention threshold T is away from the mean μ using the formula:

$$k = \frac{T-\mu}{\sigma} \quad (3)$$

2. Apply Chebyshev's inequality so as to find the upper bound probability of exceeding the threshold:

$$P(X \geq T) \leq \frac{1}{k^{\wedge}2} \quad (4)$$

This approach provides a conservative estimate of the probability that the individual loss will exceed the retention threshold.

The inequality gives the upper bound for the probability that the loss exceeds the retention level *T*. By adjusting *T* (the retention threshold), insurers can balance their risk exposure. Higher retention levels result in a smaller probability of exceeding the threshold, but they also expose the insurer to a greater risk. In reinsurance, this method allows the insurer to quantify the likelihood of extreme losses, which helps in setting an appropriate retention level that aligns with their risk tolerance.

The Monte Carlo Simulation is a numerical approach that estimates the probability of exceeding retention by simulating random losses based on the observed distribution. This method generates a lot of random samples and calculates the proportion of the simulated losses that exceed the retention threshold, which gives the estimated probability $P(X \geq T)$ or the probability that the insurer's loss exceeds the retention level. The process for applying the Monte Carlo simulation is as follows:

1. The data input. Use historical data for individual losses (e.g. the losses from 2018 to 2022). Future losses are assumed to follow the same distribution as the historical data.
2. Generate random samples. Create a lot of random samples from the observed historical data. Each sample represents a simulated individual loss.
3. Determine exceedances. For each simulation, check whether the simulated loss exceeds the retention threshold *T*. The threshold T is typically calculated utilizing the following formula:

$$T = \mu + k\sigma \quad (5)$$

4. Probability estimation. Estimate the probability $P(X \geq T)$ by calculating the ratio of the number of the simulations where $X \geq T$ to the total number of the simulations run. In other words, the probability that the loss exceeds the retention threshold is estimated as the ratio of the number of the samples that exceed the threshold to the total number of the simulations run:

$$P(X \geq T) = \frac{Number\ of\ simulations\ where\ X \geq T}{Total\ number\ of\ simulations} \quad (6)$$

By repeating this process for a lot of simulations (e.g. 10,000 or more), the Monte Carlo simulation provides a more accurate estimate of the retention threshold exceedance probability, thus reflecting the inherent uncertainty and variability in the loss data.

Normal Distribution assumes that individual losses follow normal distribution, characterized by the mean (μ) and the standard deviation (σ). While this is a simplifying assumption, it is commonly used in insurance modelling when data are expected to follow the bell-shaped curve.

To calculate the retention threshold exceedance probability in a normal distribution, these steps are to be followed:

1. Calculate the *Z*-score. The *Z*-score measures how many standard deviations the retention threshold *T* is away from the mean μ. The Z-score is given by:

$$Z = \frac{T-\mu}{\sigma} \quad (7)$$

where:

μ is the mean individual loss,

σ is the standard deviation of the individual losses, and

T is the retention threshold.

2. Determine the probability. Once the Z-score is calculated, the standard normal distribution table (or the computational tool) can be used to find the cumulative probability $P(Z)$, which represents the probability that a loss is less than the threshold. The probability of exceeding the threshold is the complement of this cumulative probability:

$$P(X \geq T) = 1 - P(Z) \tag{8}$$

where $P(Z)$ is the cumulative probability corresponding to the Z-score.

Although this method assumes that the losses are symmetrically distributed around the mean, real-world data, particularly in the case of motor casco insurance, may not perfectly follow a normal distribution. Loss distributions in insurance often exhibit skewness or heavy tails, which means that extreme losses are more frequent than predicted by a normal distribution. Despite this limitation, the normal distribution approach provides a convenient and broadly used method for estimating the probability of exceedances under the assumption of normality.

Each of the three methods offers a different way to estimate the retention threshold exceedance probability. Chebyshev's inequality provides a conservative estimate that does not rely on the assumptions about the distribution of losses, making it useful when the distribution is unknown. The Monte Carlo simulation uses historical data to simulate potential future losses, offering a flexible and data-driven approach. Finally, normal distribution assumes a bell-shaped distribution of losses, which simplifies the calculation but may not always reflect the true distribution of losses in real-world insurance data.

By applying these methods, it is possible to estimate the retention threshold exceedance probability under different assumptions, which helps insurers make more informed decisions on their risk retention levels.

RESULTS AND DISCUSSION

The data presented in Table 1 reflect the trends in the number of claims, the paid losses, and the mean losses in casco insurance over a five-year period from 2018 to 2022.

Several key trends and fluctuations can be observed over the five-year period. The number of claims has shown a gradual decrease from 407 760 in 2018 to 352 396 in 2022. This decline may reflect a decrease in the total number of insured vehicles or an improvement in risk management and underwriting.

The paid losses, however, did not follow the same downward trend. While the losses remained relatively stable between 2018 and 2021, they increased significantly in 2022, reaching BGN 380 201 243, which is substantially higher than in previous years.

The mean loss per claim steadily increased from BGN 817 in 2018 to BGN 1,079 in 2022, which indicates that, while the total number of claims decreased, the cost

Table 1 The number of claims and the paid losses in motor casco insurance (in BGN) by year

Year	Number of claims	Paid losses (BGN)	Mean loss (BGN)
2018	407 760	333 303 426	817
2019	403 189	345 203 045	856
2020	369 373	341 883 686	926
2021	351 763	339 662 479	966
2022	352 396	380 201 243	1079
Total mean			929
Standard deviation			102

Source: Authors, based on Financial Supervision Commission (https://www.fsc.bg/en/)

per claim had been rising over the years. In particular, the sharp rise in 2022 (from BGN 966 in 2021 to BGN 1,079) may suggest a change in the nature of the claims, such as higher costs associated with repairs, medical expenses, or legal settlements.

As for the fluctuations and variability, the standard deviation of BGN 102 calculated from the mean loss indicates a moderate variability in the annual loss amounts. The fluctuations in the paid losses between 2018 and 2022, especially the marked increase in 2022, demonstrate that, while there is a general trend of rising costs, the year-to-year differences can be significant. These fluctuations could be driven by various factors, such as changes in the frequency of catastrophic claims, inflation, or shifts in claim handling processes.

Overall, the data suggest that, while the number of claims has been decreasing, the cost per claim is on the rise, which could pose a greater financial burden on insurers. The significant rise in the paid losses in 2022 warrants further investigation into the underlying causes, such as more expensive claims or unforeseen events that led to higher payouts. Overall, while the number of claims shows some stability, the large variations in the loss amounts and the mean losses emphasize the unpredictable nature of the risk involved, underlining the importance of effective risk assessment and retention strategies.

Based on the given data, this study suggests that the direct insurer's retention threshold should be set at BGN 1 115 per individual claim, which is derived from the mean loss of BGN 929 plus 20% of the mean (i.e. BGN 186).

This threshold represents a strategic point where the insurer retains a significant but not excessive amount of risk, transferring the remaining risk to reinsurance. In assessing how accurate risk assessment is for determining this threshold, the following three methods can be utilized, namely Chebyshev's inequality, the Monte Carlo simulation, and the normal distribution approach.

Chebyshev's inequality is a non-parametric method that provides a bound on the probability of extreme outcomes, regardless of the distribution of the data. This method would estimate the probability that a claim will exceed the retention threshold (BGN 1 115) based on the mean and standard deviation of the data. Given the fact that Chebyshev's inequality applies to any distribution, it is more conservative and would likely provide an upper bound on the likelihood of exceeding the threshold. However, it may not offer the level of precision needed for more informed retention decisions because it tends to overestimate the probability in comparison to methods that take the actual data distribution into account.

By contrast, the Monte Carlo simulation is much more flexible and powerful. It simulates many possible future loss scenarios based on historical data and random sampling, generating a probability distribution for losses. Given the data, Monte Carlo simulations would estimate the retention threshold exceedance probability by running thousands of simulations with varying assumptions for claim severity and frequency. This method would offer a more precise estimate of risk and help the insurer better understand the likelihood of extreme events. It is particularly valuable in assessing complex, uncertain risk profiles and would provide a more tailored and accurate estimate than Chebyshev's inequality.

Assuming that a loss distribution follows a normal distribution, the insurer could calculate the retention threshold exceedance probability by considering the mean and the standard deviation. Based on the data, the normal distribution approach could estimate the percentage of the claims that would likely exceed the BGN 1 115 threshold. However, normal distribution is limited by the assumption that losses follow a bell-shaped curve. If the actual distribution of losses is skewed or has fat tails (as is often the case in motor casco insurance), this method may underestimate the probability of extreme losses. Despite its simplicity, the normal distribution approach could still provide a useful baseline estimate, particularly in the absence of more complex data modelling.

Table 2 presents the probabilities of exceeding the retention threshold of BGN 1115 calculated using the three different methods: Chebyshev's Inequality, the

Monte Carlo Simulation, and Normal Distribution. The calculations were performed using Microsoft Excel.

According to Chebyshev's Inequality: the 24.52% retention threshold exceedance probability represents a very conservative estimate. Chebyshev's inequality guarantees that no more than this percentage of claims will exceed the threshold, but because it does not make assumptions about the data distribution, it tends to overestimate the risk. While useful in the absence of detailed information about the loss distribution or when data is limited, it is likely to overstate the risk of large claims exceeding the threshold in this case.

Table 2 The retention threshold exceedance probabilities calculated using the three methods

Method	Retention threshold (BGN)	Retention exceedance probability
Chebyshev's Inequality	1 115	24.52 %
Monte Carlo Simulation	1 115	1.99 %
Normal Distribution	1 115	2.16 %

Source: Authors

The result of the 1.99% probability calculated from the Monte Carlo simulation is a more realistic estimate. By simulating 10000 random outcomes based on the observed loss data, this method captures the underlying variability of the losses. It can model the non-normal features of the loss distribution (such as skewness or heavy tails, providing a more precise risk estimate, which takes into account the full range of potential outcomes and reflects a more accurate risk assessment for the insurer).

The 2.16% probability calculated from the normal distribution approach is very close to the Monte Carlo result, which suggests that, for this particular dataset, normal distribution is a reasonable approximation of the actual loss distribution. Despite the moderate skewness perceived in the data, the normal distribution approach provides a useful and efficient tool for estimating retention thresholds when data are roughly symmetric or near normal by their nature. The close alignment between this method and the Monte Carlo simulation indicates that, in this particular case, the assumption of normality is valid enough to provide a similar estimate.

The results from the above calculations of the retention threshold exceedance probabilities allow for the following summary:

1. The conservatism of Chebyshev's Inequality. The method offers a safe, conservative approach but lacks precision, especially for extreme loss probabilities. The high retention threshold exceedance probability (24.52%) calculated utilizing Chebyshev's inequality emphasizes the method's conservative nature, not taking into consideration the actual characteristics of the loss data, thus leading to an overestimation of risk and to potentially higher reinsurance costs.

2. More accurate assessment by applying the Monte Carlo Simulation and the Normal Distribution Approach. The Monte Carlo simulation and normal distribution approaches provide very similar results, with the probabilities of 1.99% and 2.16%, respectively. These methods provide a more precise and realistic estimate of the retention threshold exceedance risk. The small difference between these two methods suggests that the assumption of normality is reasonably accurate in this case. Therefore, an insurance company can be more confident in using these methods for risk assessment.

3. The reliability of the Monte Carlo Simulation. The Monte Carlo simulation is generally considered the most flexible and accurate method for complex risk assessment. The relatively low probability it provides (1.99%) gives a more realistic view of the risk of loss and can be used for better-informed decision-making when setting retention thresholds. However, this approach is computationally intensive and requires a substantial amount of data and simulation runs.

4. Given the large discrepancy between Chebyshev's Inequality and the other two methods, the Monte Carlo simulation would be the most reliable approach for setting the retention threshold, especially when

dealing with complex, non-normal distributions. The normal distribution approach could also be used as a simpler alternative, but its assumptions about the data distribution should be carefully considered.

5. An insurance company can be confident in using the retention threshold of BGN 1115 based on the lower probabilities obtained through the Monte Carlo and normal distribution methods. This threshold effectively balances risk retention with risk transfer, based on the relatively low (around 2%) probability of exceeding it.

A sensitivity analysis was performed to validate the robustness of the results and assess how changes in key assumptions, such as the mean loss, impact the retention threshold exceedance probability. The purpose of this analysis was to understand how changes in the mean loss (μ) affect the retention threshold exceedance probability (BGN 1,115). The examination of this relationship can allow for the assessment of the impact of varying loss scenarios on the risk profile and provide a further context for the calculated probabilities. The sensitivity analysis helps illustrate the potential variations in the risk that may arise from changes in the loss levels, thereby supporting the results obtained using the Monte Carlo and normal distribution based on the assumptions about the underlying distribution of the losses.

The results presented in Figure 1 after the calculations have been made clearly demonstrate the fact that, as the mean loss increases, so does the retention threshold exceedance probability. When the mean loss is BGN 900, the threshold exceedance probability is 0.0175 (1.75%), which suggests that, with lower mean losses, there is a very small chance of exceeding the threshold, and thus the need for reinsurance remains minimal. As the mean loss reaches BGN 1,100, the probability increases to 0.4415 (44.15%), indicating a significantly higher risk of exceeding the retention threshold, which highlights the increasing likelihood that the insurer will need to transfer risk to reinsurance if the average loss per claim becomes substantial.

These findings underscore the importance of closely monitoring mean loss levels. As the mean loss approaches or exceeds the retention threshold (BGN 1,115), the likelihood of exceeding it rises sharply, suggesting the need for increased reinsurance coverage.

The Monte Carlo simulation generated a probability of 1.99% based on realistic, random variations in claim amounts and frequencies. When comparing this to the results obtained from the sensitivity analysis, the following can be perceived: at the mean loss of BGN

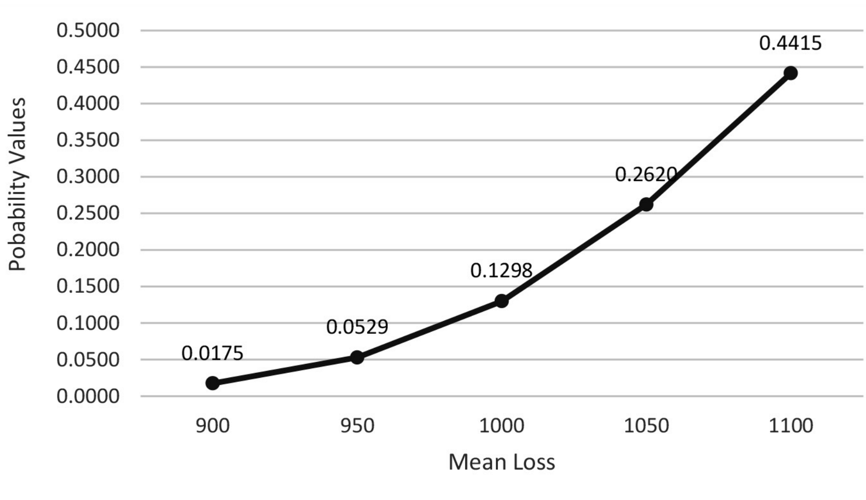

Figure 1 The changes in the retention threshold exceedance probability with the increased mean loss

Source: Authors

950, the threshold exceedance probability is 0.0529 (5.29%). For the mean loss values around BGN 1,100, the probability increases significantly, ranging from 0.2620 (26.20 %) to 0.4415 (44.15%).

Given this, the 1.99% probability derived from the Monte Carlo simulation lies in the lower end of the sensitivity analysis curve, corresponding to a mean loss between BGN 900 and 950, which suggests that the Monte Carlo result represents a scenario where the average claim loss is relatively low, and the threshold exceedance probability remains small. Thus, the Monte Carlo simulation provides a more precise estimate of risk for lower loss scenarios, and the sensitivity analysis supports this finding by showing how small variations in the mean loss directly affect the likelihood of exceeding the threshold.

CONCLUSION

The comparative analysis of the three statistical methods - Chebyshev's Inequality, the Monte Carlo Simulation, and the Normal Distribution - estimates the retention threshold exceedance probability in motor casco insurance. The analysis is exclusively based on the official, aggregated data published by the Bulgarian Financial Supervision Commission for the period from 2018 to 2022. The results indicate that the Monte Carlo simulation offers the most accurate and reliable estimates of retention-level exceedance probabilities, primarily due to its flexibility in modelling complex and non-normal loss distributions. The normal distribution method produced the results that closely align with those of the Monte Carlo Simulation, suggesting a reasonable approximation for the dataset under consideration. In contrast, Chebyshev's Inequality yielded significantly higher and more conservative estimates, reflecting its generalized, non-parametric nature.

The practical significance of these findings lies in their direct applicability to real-world insurance operations. The ability to more accurately estimate the retention threshold exceedance probability is critical for optimizing risk transfer strategies, determining reinsurance structures, and ensuring capital adequacy. Given the substantial role of motor casco insurance within the Bulgarian non-life insurance market, the implementation of more precise risk assessment methods based on available official data can contribute to more efficient financial planning and enhanced sector stability.

Despite the methodological robustness, this research is subject to several limitations. First, the study is constrained by the aggregated market-level data used, since detailed insurer-level information is not publicly available. As a result, the analysis cannot account for the heterogeneity in underwriting practices, risk profiles, or claim management strategies across individual insurers. Second, the examined timeframe is limited to a five-year period due to the absence of longer historical datasets in the public domain, which restricts the ability to assess long-term trends and may limit the generalizability of the findings. Third, all the calculations were made using Microsoft Excel, which, while suitable for the scope of this research, does not offer the computational depth of specialized statistical or actuarial pieces of software. Finally, although other advanced risk modelling techniques such as machine learning or multivariate simulation could enhance the predictive performance, their application is not feasible in this context due to the restricted availability of detailed input data.

Building on the current findings, future research could be conducted to access the insurer-level data that would allow for a better understanding of risk variability and retention behavior across companies. Furthermore, incorporating longer time series or event-based stress testing could enhance the evaluation of tail risk and extreme loss scenarios. More available data could allow for the application of advanced analytical methods, including machine learning algorithms, stochastic modelling, or copula-based approaches, and provide deeper insights into loss dependencies and risk concentration. Additionally, expanding the scope to include multi-line insurance portfolios would offer a broader view of retention optimization at the portfolio level. Finally, future studies may consider the impact of macroeconomic or regulatory changes on frequency and severity of losses, further informing the dynamic adjustment of retention strategies.

In conclusion, this research supports the use of the Monte Carlo simulation as one of the most suitable methods among the considered ones for determining optimal retention levels in motor casco insurance based on officially available data. It offers a practical framework for insurers operating under data constraints.

REFERENCES

Bulut Karageyik, B., & Şahin, Ş. (2017). Determination of the Optimal Retention Level Based on Different Measures. *Journal of Risk and Financial Management*, *10*(1), 4. https://doi.org/10.3390/jrfm10010004

Cai, J., & Tan, K. (2007). Optimal retention for a stop-loss reinsurance under the VaR and CTE risk measures. *ASTIN Bulletin*, *37*(1), 93-112. https://doi.org/10.2143/AST.37.1.2020800

Chukwudum, O. (2019). Reinsurance pricing of large motor insurance claims in Nigeria: An extreme value analysis. *International Journal of Statistics and Probability*, *8*(4). https://doi.org/10.5539/ijsp.v8n4p1

Covrig, M., & Badea, D. (2017). Some generalized linear models for the estimation of the mean frequency of claims in motor insurance. *Economic Computation and Economic Cybernetics Studies and Research*, *4(51)*, 91-107.

Djuric, Z. (2013). Collective risk model in non-life insurance. *Economic Horizons*, *15*(2), 167-175. https://doi.org/10.5937/ekonhor1302163dD

Erusalimov, R., & Iliev, N. (2022). The impact of COVID-19 on development of motor casco insurance in Bulgaria. *Business Management*, *20(1)*, 35-45.

Fu, L., & Wang, H. (2021). Estimating insurance attrition using survival analysis. *Variance: Advancing the Science of Risk*, *8*(1), 55-75.

Glineur, F., & Walhin, J. (2006). De Finetti's retention problem for proportional reinsurance revisited. *Blätter Der DGVFM*, *27(3)*, 451-462. https://doi.org/10.1007/BF02809170

Heinrich, T., Sabuco, J., & Farmer, J. (2021). A simulation of the insurance industry: The problem of risk model homogeneity. *Journal of Economic Interaction and Coordination*, *17(2)*, 535-576. https://doi.org/10.1007/s11403-021-00319-4

Hu, X., Yang, H., & Zhang, L. (2015). Optimal retention for a stop-loss reinsurance with incomplete information. *Insurance: Mathematics and Economics*, *65*, 15-21. https://doi.org/10.1016/j.insmatheco.2015.08.005

Kaishev, V. K. (2004). Optimal retention levels, given the joint survival of cedent and reinsurer. *Scandinavian Actuarial Journal*, *2004*(6), 401-430. https://doi.org/10.1080/03461230410020437

Kočović, J., Mitrašević, M., Kočović, M., & Jovović, M. (2011). Capital allocation problems of non-life insurance companies. *Economic Horizons*, *13*(2), 45-69.

Kroll, Y., & Nye, D. (1991). Reinsurance retention levels for property/liability firms: A managerial portfolio selection framework. *Insurance: Mathematics and Economics*, *10*(2), 109-123. https://doi.org/10.1016/0167-6687(91)90004-H

Leiria, M., Rebelo, E., & deMatos, N. (2021). Measuring the effectiveness of intermediary loyalty programmes in the motor insurance industry: Loyal versus non-loyal customers. *European Journal of Management and Business Economics*, *31*(3), 305-324. https://doi.org/10.1108/ejmbe-05-2020-0103

Li, S., Zhu, Z., & Peng, J. (2022). Optimal retention for uncertain stop-loss reinsurance model. *Journal of Uncertain Systems*, *15*(4). https://doi.org/10.1142/s1752890922430024

Noviyanti, L., Soleh, A. Z., Chadidjah, A., & Rusyda, H. A. (2018). Optimal Retention for a Quota-Share Reinsurance. *Jurnal Teknik Industri: Jurnal Keilmuan Dan Aplikasi Teknik Industri*, *20*(1), 25-32. https://doi.org/10.9744/jti.20.1.25-32

Roos, E., Brekelmans, R., van Eekelen, W., den Hertog, D., & van Leeuwaarden, J. S. H. (2022). Tight tail probability bounds for distribution-free decision making. *European Journal of Operational Research*, *299*(3), 931-944. https://doi.org/10.1016/j.ejor.2021.12.010

Shahriar, B., & Ahmadi, S. M. M. (2008). The calculation of optimal reinsurance retention by value at risk approach. *Journal of Economic Research*, *8*(28), 223-243.

Soleh, A. Z., Noviyanti, L., & Nurrahmawati, I. (2015). Retention for stoploss reinsurance to minimize VaR in compound Poisson-Lognormal distribution. *AIP Conference Proceedings, 1692*, 020026. https://doi.org/10.1063/1.4936454

Tsanakas, A., & Millossovich, P. (2015). Sensitivity analysis using risk measures. *Risk Analysis, 36*(1), 30-48. https://doi.org/10.1111/risa.12434

Xiong, L., & Hong, D. (2020). Using Monte Carlo simulation to predict captive insurance solvency. In *Proceedings of the 2020 4th International Conference on Compute and Data Analysis* (pp. 84-88). New York (NY): Association for Computing Machinery.

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